BluePrintDemo

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January 28, 2026

Chapter 1

Sum Formulas

This chapter contains classical summation formulas that serve as a demonstration of the Lean Blueprint workflow.

1.1 Sum of Natural Numbers

Theorem 1 (Sum of Natural Numbers)**.** *For all* $n \in \mathbb{N}$*:*

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

Proof. By induction on n. The base case $n = 0$ is trivial since both sides equal 0. For the inductive step, assuming the formula holds for n, we have:

$$\sum_{i=0}^{n+1} i = \sum_{i=0}^{n} i + (n+1) = \frac{n(n+1)}{2} + (n+1) = \frac{(n+1)(n+2)}{2}$$

which is the formula for $n + 1$. □

1.2 Sum of Squares

Theorem 2 (Sum of Squares)**.** *For all* $n \in \mathbb{N}$*:*

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Proof. By induction on n. The base case $n = 0$ is trivial. For the inductive step, assuming the formula holds for n:

$$\sum_{i=0}^{n+1} i^2 = \sum_{i=0}^{n} i^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2$$

Simplifying:

$$= \frac{n(n+1)(2n+1) + 6(n+1)^2}{6} = \frac{(n+1)(2n^2 + n + 6n + 6)}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$$

which is the formula for $n + 1$. □